

$$\left[ \begin{array}{cccc|c} 0 & 4 & -8 & 0 & 4 \\ 2 & 6 & -6 & -2 & -4 \\ 2 & 7 & -8 & 0 & -1 \end{array} \right] \xrightarrow{\text{row reduction}} \left[ \begin{array}{cccc|c} 1 & 0 & 3 & 0 & -4 \\ 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array} \right]$$

**Definition**

A *pivot position* in a matrix is a position that after row reduction contains a leading one.

A *pivot column* of a matrix is a column that contains a pivot position.

**Theorem**

- 1) A system of linear equations is inconsistent if and only if the last column of its augmented matrix is a pivot column.
- 2) Free variables of the system correspond to non-pivot columns of the coefficient matrix.
- 3) The system has only one solution if and only if every column of its augmented matrix is a pivot column, except for the last column.

## Theorem

A system of linear equations can have either 0, 1, or infinitely many solutions.

*Proof.*

